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| Extended Kalman Filtering of State and Sensor Bias Estimation of a Li-Ion Battery Model |
| MAE 298 – Estimation Theory Final Project |
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| Abstract |
| *The increasing demand for electric vehicles (EVs) has led to technological advancements in the field of battery technology. State of charge (SOC) estimation is a vital function of the battery management system - the heart of electric vehicles, and Kalman filtering is a common method for SOC estimation. Due to the non-uniformities in tuning and testing scenarios, quantifying performance of SOC estimation algorithms is difficult. In this work, an SOC estimation algorithm is developed, Extended Kalman Filter (EKF), and tested for a variety of scenarios like adding sensor noise and bias to terminal voltage and current, and varying state and parameter initializations. In addition, a Dual EKF is implemented to estimate the voltage and sensor bias and compared against the State EKF for robustness to sensor bias/noise.* |

Table of Contents

**1. Introduction & Literature Review** **2**

1.1 Motivation 2

1.2 Background3

1.3 Objectives3

**2. System Modeling & Analysis 4**

2.1 Overview of Li-Ion Battery5

2.1.1 State of Charge5

2.1.2 Open Circuit Voltage5

2.2 Electrical Equivalent Circuit Model5

2.1.1 Continuous Time Model5

2.1.2 Discrete Time Model5

2.3 Sensor Bias Modeling5

2.3.1 Current Sensor Bias5

2.3.2 Voltage Sensor Bias5

2.3.3 Observability Analysis5

**3. Algorithms & Implementation 4**

3.1 Linear Kalman Filter5

3.2 Extended Kalman Filter5

3.3 State & Parametric Estimation5

3.3.1 Dual EKF Estimation5

**4. Results & Discussion 4**

4.1 The Setup5

4.1.1 Simulation Setup5

4.1.2 Performance Indices5

4.2 Simulation Results5

4.2.1 State KF vs EKF5

4.2.2 State EKF vs Dual EKF 5

4.2.3 Sensor Bias Estimation5

**5. Future Work4**

# 1. Introduction

## Motivation

In recent years, high importance has been placed on the stress levels that technology puts on the environment. This factor has created an increasing demand for electric vehicles that can be part of an eco-friendly solution.

In an electric vehicle, batteries store the electrical energy in an electrochemical reaction for later use. There are several types of batteries in the industry; the most popular are lead-acid, nickel, alkaline and lithium-ion []. Lithium has become very popular because it is light metal, has the greatest electrochemical potential and provides the largest specific energy per weight. Current lithium-ion battery technology allows EV to cover about 180-350 km per battery charge [].

The harsh operating conditions in EVs necessitates for a system to protect, monitor and control the batteries. Such a system is called the Battery Management System (BMS). Among several key functions of the BMS, one particular function, namely State of Charge (SOC) estimation is investigated in this paper.

SOC estimation is equivalent to the fuel gauge of an IC engine vehicle but unlike the fuel level, SOC cannot be measured directly as it depends on the concentration of lithium ions at the electrodes []. Moreover, due to the differences among the cells, finding SOC for a whole battery pack can be challenging. This motivates the use of algorithms capable of estimating SOC accurately and reliably using other measurable quantities. Two of the challenges for reliable SOC estimation included in this paper are noisy and bias sensor measurements and variation of battery parameters. Other challenges such as variations in temperature and battery aging as well as the overall complex and nonlinear behavior of batteries are not being considered in this work. Model-based estimation techniques, specifically Kalman filtering is used in this paper to estimate SOC. The objective of this paper is to develop an algorithm to estimate SOC, sensor bias, and to test its performance under several operating conditions.

## Background

It has always been a big concern to estimate the SOC for energy storage devices. The estimation accuracy of SOC does not only give an information about the remaining useful capacity, but also indicates the charge and discharge strategies, which have a significant impact on the battery. Thus, a Li-ion battery may have different capacities due to aging, ambient temperature and self-discharge effects.

There are three primary methods of SOC estimation:

* *Current-based methods* use the ‘Coulomb counting’ equation relating the current drawn from (or supplied to) the battery and its capacity to estimate SOC.
* *Voltage-base methods* use the relationship between open-circuit voltage and SOC.
* *Model-based estimation* uses mathematical models to relate measured signals like terminal voltage to SOC and is known to give accurate and precise estimate []

Model-based estimation is used in this paper and has two distinct sub-problems, namely the mathematical model of the battery and the estimation algorithm.

## Objectives

The objective of this thesis is to develop a SOC estimation algorithm for a Li-ion battery modeled as a second-order RC equivalent circuit model and to test its performance under different operational scenarios, such as

* Sensor noise variation
* Sensor bias variation
* Parameter variation
* Sensor bias estimation

# 2. System Modeling & Analysis

## Overview of Li-Ion Battery

By definition batteries are electro-chemical devices, meaning that they generate an electric potential due to the interaction of differing elements and chemistries when placed in very specific proximity to each other. The net result is that the physical construction and chemistry of a battery dominates the electrical, safety, and physical properties of the battery.

Due to its high capacity, lightweight, and ability to be recharged effectively, Lithium-Ion batteries have been the subject of continued research and technical improvement throughout the past decade. Its electrical properties arise due to diffusion of lithium ions and electrons through an electrolyte between the anode and the cathode of the battery.

This description is admitted simplistic but it provides enough detail to immediately suggest that the first principle model of this battery would contain partial differential equations. In actual fact, it turns out to be correct that batteries contain PDE behavior, and are therefore of infinite dimension computationally. In a classical and modern sense, engineering techniques predicated on solving partial differential equations are computationally expensive and non-trivial to setup correctly.

This distinction is important highlight, since the objective of this paper is to develop a tractable numerical estimator for the internal states of the battery. While a PDE description of the battery truly reflects its accurate and complex behavior, it fails to be a useful tool to create a representative model of the battery that captures the salient dynamics of the system without inducing an enormous computational overhead and unrequired complexity.

In response to these previous comments, this paper seeks to develop and demonstrate a viable lumped parameter battery model with enough fidelity to accurately estimate the internal states of the battery without inducing a significant computational burden.

## State of Charge

As mentioned above, a Li-Ion battery is a complex system. In order to develop a simplified model that is both detailed ‘enough’ and numerically tractable, it is important to quantify important internal states of the battery other than the terminal voltage and the current, which are the only two directly measurable electrical properties of the battery.

It turns out that one of the most important quantities of a battery is its State of Charge or SOC. The SOC provides a measure of the battery’s available capacity, effective functioning as the fuel gauge of the battery. This is an important parameter to know since the safety of many batteries, such as the Li-Ion variety used in this paper, have the potential to be extremely dangerous and even explode or cause fires if mishandled, overcharged, or over-discharged.

Unfortunately, the SOC is not a directly measurable quantity and must be estimated in order to make available for application in controls or battery management systems. To address this innate drawback, this paper presents the Extended Kalman Filter as the estimation technique of choice to reliably and accurately predict the SOC of the battery of interest.

Even though the instantaneous SOC of the battery is not measurable, a model of its dynamics is easily developed and formulated as the amount of current capable of being sourced from a finite maximum capacity battery. Mathematically, this can be written as follows.

(1)

It should be noted that the rate of change of the SOC can approximately be considered proportional to the current in or out of the battery, normalized by the capacity of the battery. In many cheap battery management systems, the SOC is approximated by numerically integrating the current. While this approach is attractive in theory, in reality this technique, called Coulomb Counting, diverges extremely quickly from the actual SOC of the battery. Therefore, the failure of this type of “open loop” estimation leads to the development of closed loop estimation schemes explained in more detail later in the paper.

## Open Circuit Voltage

In addition to the State of Charge of a battery, another key quantity of interest is its ‘Open Circuit Voltage’ or OCV. By definition this is the no-load voltage measured after the battery settled into a state of internal equilibrium.

Without diving into exact detail, it is logical to presume that there might exist some quantifiable relationship between the SOC and the OCV. In fact, it has been experimentally verified that the OCV of a battery is a function of the batteries instantaneous SOC. This fact becomes extremely useful later when developing a lumped parameter battery model, shown in the next section.

While determining an accurate relationship between SOC and OCV requires very precise laboratory equipment and highly control testing environments, it is possible to characterize a battery with a specific OCV versus SOC curve. For a typical Li-Ion battery, this OCV/SOC relationship is typically nonlinear. For the purposes of this paper, the experimental relationship between these two quantities are assumed to be nonlinear and treated as a fully known relationship from a lookup table. The figure below shows the lookup table data in a graph employed in this paper.



## Electrical Equivalent Circuit Model

As previously covered in detail, the predominate issue with controlling or estimating states defined by complex models such as PDEs is the inability to compute solutions without expansive computational resources and time. In the domain of real-time engineering systems, these models do not meet the cost/performance trade off required to make them a viable solution. Instead, it is desired to develop a finite dimensional model capable of matching the dominate dynamics of the system, and whose implementation cost is within reason. In the realm of system modeling, this typically suggests a lumped parameter model. The benefits of a such a model is the ability to capture large dynamics while maintaining computationally inexpensive.

To bypass this problem, it is desired to use simplified low order dynamic models that are numerically tractable for the intended application. This leads to the use of “Equivalent Circuit Models” (ECMs). The benefit of ECMs is their inherent ease of derivation and application which becomes apparent in commercial uses where computing overhead is extremely limited, for cost considerations.

Although not explicitly mentioned above, the relationship between the SOC and OCV hints at a method to encapsulate the effect of the SOC (non-measurable) within a measurable quantity, the OCV. Furthermore, as nonideal electrical device any battery must exhibit an ‘equivalent series resistance’ or ESR. When taken together these two observations suggest building an electrical circuit. However, an equivalent battery model utilizing just these two relationships would fail to capture time varying dynamics. To address this short coming, resistor/capacitor (RC) branches can be added as the current through a capacitor is function of the time rate of change of the voltage across the capacitor, thereby adding time varying dynamics to the model of a general battery “circuit.”

Once the circuit is setup as shown in figure(????), it becomes clear how the OCV, ESR, and RC pairs can be used to model the terminal voltage of the battery under test.



(soc)

This paper will use the “Dual Polarity” (DP) equivalent circuit model as it is not only one of the most popular Li-Ion battery models in commercial use today but also the relative ease of reframing the model into an Extended Kalman Filter (EKF) algorithm.

The circuit schematic for the DP model is shown above. Notice that the terminal voltage of the battery is easily shown to be related to the dynamics of the open circuit voltage , the series resistance and the two RC circuits. By application of basic circuit rules (Kirchhoff’s current and voltage laws), the dynamics of the system can easily be derived as shown in the next few sections.

## Continuous Time Model

As mentioned previously, by using standard circuit analysis, we can extract the dynamic behavior of the system. By applying KVL around the complete loop of the circuit, we get the following expression for the terminal voltage of the circuit as a function of the internal elements of the circuits

(2)

By applying KCL to both RC branches we derive the following equations

(3)

(4)

By including the expression for SOC with the equations defined above, the continuous time state space model can be written as follows

(5)

(6)

A noticeable feature of this state space is the linear behavior of the state equations and the nonlinear behavior of the output equations. Therefore, the system is inherently nonlinear, indicating that estimating the SOC for the nonlinear model would most likely require at the very least an Extended Kalman Filter (EKF) or even more advanced methods use as the Unscented Kalman Filter (UKF) for obtaining an accurate estimation.

## Discrete Time Model

While the continuous time model is an important start in the process of estimating the system, in today’s computer age, it is significantly easier to implement discrete time models on modern computers which inherently are limited to finite numerical representations of numbers.

To transform the continuous time model into a discrete time state space, the closed form discretization formulas (shown below) were applied the appropriate matrices and vectors of the continuous time model to produce the following discrete state and output equations.

(6)

## Sensor Bias Modeling

When setting up an estimation problem, it is important to incorporate physical phenomena that exist under real world conditions whenever possible. One such condition that is common is suboptimal sensor measurements being used as inputs into estimators. The predominate means through which this is exemplified is the presence of a measurement bias or offset. Typically, the bias of a sensor is calibrated for or at least guaranteed accurate within some specified tolerance. Since this bias will always be present and may change with time, it is useful to estimate the value of these biases.

In this paper, a Dual Extended Kalman Filter is implemented to estimate the terminal voltage and current sensor bias along with the state of charge using an augmented state-space model involving the bias. Bias is modeled as random-walk where, it is assumed to be essentially constant but is capable of varying slowly over time, driven by some process modeled using a small fictitious noise. Both voltage and current sensor bias are considered independently in this paper.

In order to generate data that exhibits a bias, the current and voltage biases were included in either the input current data or the terminal voltage measurement respectively, as constant offsets. This provides known biases which facilitate initial validation of the bias estimation scheme as well as provide some means of tuning the estimator for future testing.

To represent sensor bias in current and voltage, equations (8) and (8) are used, and the augmented state-space model is modified as well as the output terminal voltage equation. and represent the bias in the current and voltage sensors, respectively, and and represent the corresponding measured (inaccurate) signals.

(8)

(9)

## Current Sensor Bias

The continuous time state-space model of the system with current sensor bias can be obtained by substituting Equation (8) in Equations (1) to (5). This augmented model has 4 states. The output is the same terminal voltage but the input is now the measured current (current signal with bias).

(10)

The augmented state space and output terminal voltage equations (in discrete time) then become

(11)

(12)

Where is the augmented current sensor bias included as a state.

## Voltage Sensor Bias

Similar to the case with current sensor bias, the continuous time state-space model of the system with voltage sensor bias can be obtained by substituting Equation (9) in Equations (1) to (5). The resulting augmented model also has 4 states. The input is the current but the output is now the measured terminal voltage .

(13)

The augmented state-space and output terminal voltage equations (in discrete time) then become

(14)

(15)

Where is the augmented voltage sensor bias treated as a state. Also, notice that is a nonlinear term which in order to form the state space model it was included to the left of the equation and is also on the left. These two terms can then be added back in the EKF algorithm using a lookup table that provides the nonlinear relation between and (in the case of ) and the bias can just be added back as a time update term.

## Observability Analysis

# 3. Algorithms & Implementation

do we need these plots for Lin’s lookup table data that we are using?

## Linear Kalman Filter

In the scope of academic literature, a Kalman Filter is the broad name for a class of stochastic estimation algorithms. However, in the strict sense a Kalman Filter refers to an estimation scheme for linear systems, under the assumption of additive white noise which follow a Gaussian distribution.

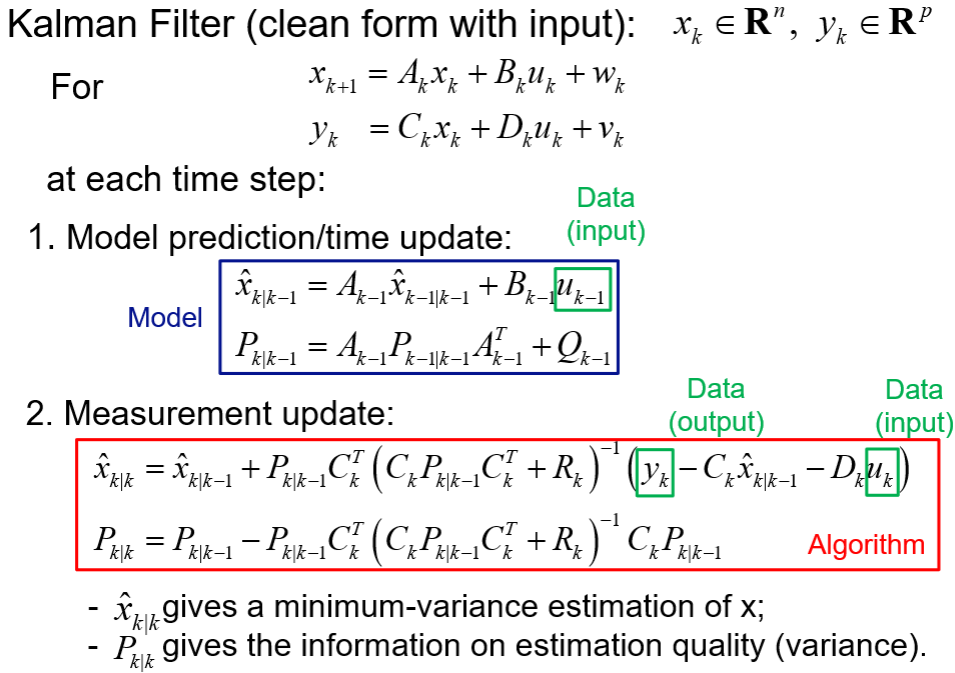
The premise of KF is to utilize the appealing properties of linear system subject to Gaussian noise, as a means of optimally estimating an unknown state of a stochastic system. There are many benefits to this approach. By assuming that the system is stochastic in nature is truer to reality than assuming a purely deterministic process. This means the computation being performed in a KF is accounting for variability and uncertainty within the system and attempts to supply an estimate of the optimal value, given this extra information. This stands in contrast to the deterministic Luenberger Observer, which uses a constant observation error gain, and does not dynamically evolve.

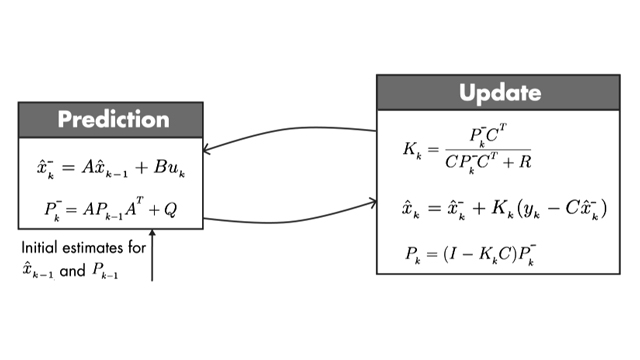
The predominate assumptions made within the formulation of the Kalman Filter are linear dynamics and additive Gaussian noises. Under these assumptions, properties of linear stochastic systems can be applied and neatly manipulated into an exact closed form algorithm.

As with many variants of Bayesian estimators, the algorithm for the Kalman Filter can be broken down into two stages.

The first stage of the Kalman Filter is called Model Prediction/Time Update. This step uses a linear model with additive Gaussian process and measurement statistics to predict the next state in the time evolution of the system.

The second stage, is called Measurement Update. This step utilizes sensor measurements of the system and statistically updated “Kalman” gain to correct the Model Prediction estimate of the next state to the most likely value. The algorithm for the Kalman Filter is shown in the figure (XXXX).



[](https://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&ved=2ahUKEwjp2uaC_sTiAhVQsZ4KHVdeAzkQjRx6BAgBEAU&url=https://www.mathworks.com/videos/understanding-kalman-filters-part-4-optimal-state-estimator-algorithm--1493129749201.html&psig=AOvVaw0RFigNvFDOek4unX-85XMG&ust=1559365079179008)

## Extended Kalman Filter

Since no true physical system can truly be considered linear, the development of an estimation scheme that works well for nonlinear systems is a practical concern.

Often in engineering practice, the knee jerk reaction to nonlinear systems is to linearize them and hope the resulting linearization errors prove to be insignificant. When this approach is applied to KF, the result is Linearized KF. Unfortunately, this algorithm typically demonstrates very poor performance since it linearizes both the dynamics of the system and covariance of the system compounding the error due to linearization.

The solution to this is the Extended Kalman Filter (EKF). EKF is an extremely important variant of the regular Kalman Filter because it is adapted to work with nonlinear systems, while requiring minimal change to the fundamental algorithm/processes. The defining difference between Linearized KF and Extended KF is that the system dynamics are not linearized. While this alternation does not seem immediately significant, it has very broad implications to the resulting performance of the estimator.

With the power of modern computing technologies, the computational limitations of numerically simulating or running nonlinear equations on microcontrollers, microprocessors, and personal computers has never been faster or cheaper. Therefore, computing a deterministic nonlinear equation is not a hindrance to most users implementing state observers. This means that instead of linearizing the dynamics of the system, the full nonlinear description of the system is preserved, reducing linearization errors.

On the other hand, nonlinear covariance and stochastic properties must be linearized since closed form transformations of nonlinear systems are extremely challenging to solve or may not even exist. Consequently, linearization of these stochastic quantities is necessary to attempt to preserve the Gaussian of the filter, even if only an approximation. Therefore, as an approximation, EKF is not an optimal observer since no guarantees of performance can made due to the need to linearize. Finally, there exists other more advance estimators such as Bayesian Estimator and Particle Filter that can give more accurate estimation results but at a tradeoff of computational expensiveness.

## State & Parameter Estimation

One of the more unique features of the Kalman filters as a class of observers, is that they can be used not only to estimate the states of the system of interest, but they can also be used to predict parameters of the system.

## Dual EKF



# 4. Results & Discussion

## The Setup

## Simulation Setup

The overall simulation setup consists of the estimation algorithms KF, EKF and DEKF used with the battery model explained previously. For this study, MATLAB is used to simulate the measurements required for the Kalman algorithms….

A typical BMS is equipped with current, voltage and temperature sensors which have limited accuracy due to intrinsic measurement noise and bias. In this work, we only considered current and voltage measurement bias. To test estimation algorithms under different sensor properties, noise and bias are added to both current as well as terminal voltage signals. The noise added is Gaussian with zero-mean and a standard deviation of 1% of the corresponding signal’s maximum value. The bias level is set as 20mV for Voltage and 12.5mA for current.

## Performance Indices

Root mean square error: RMSE is the square root of mean of square of all errors. It

is calculated using the actual and estimated values, and is computed for SOC as well

as terminal voltage. It denotes the estimation accuracy.

Infinity Norm of SOC Error: It gives the worse-case measure of the SOC error and is

given by where n is the number of samples over the whole simulation time and N is the length of the drive cycle (the sampling time is 0.1s).

Variance of SOC Error: It refers to the average variance of SOC error over whole simulation

time. Variance measures the estimate’s uncertainty and is denoted by . With every new measurement, the Kalman filter aims to reduce uncertainty and hence, the variance ideally decreases and remains constant at steady-state.

## Simulation Results

## Model Validation

A model validation was conducted to verify the derived 2nd Order Equivalent Circuit Model (ECM). This verification was performed by generating data of actual SOC and terminal voltage (with process noise) from a “true” model of a 3rd Order ECM given time and battery current data as inputs. The noise added is Gaussian with zero-mean and a standard deviation of .1% of the corresponding signal’s maximum value. The simulated “true” data was then used on a derived 3rd order circuit battery model with an EKF implementation for validation. The results are shown below.

1. b.

c. d.

**Figure X**. blah blah blah

After concluding that the derived 3rd order ECM’s SOC closely matched the “true” SOC data, a 2nd order ECM was derived and used as a framework to design an Extended Kalman Filter and Dual EKF to estimate the SOC and Voltage/Current Bias as it will be explained in the next sections.

## State KF vs EKF – SOC Estimation

A linear Kalman Filter and Extended Kalman Filter was implemented on a nonlinear equivalent circuit battery model to estimate its state of charge. Using the nonlinear data on a Linear KF gives inaccurate results and it can be seen that EKF performs a lot better. This is due to the fact that Linear KF does not consider nonlinearities of the system coming from the open circuit voltage dependence on SOC. On the other hand, EKF considers the nonlinearities of the battery model by linearization of the nonlinear model using first order Taylor series about an operating point, this in turn significantly improves the SOC estimation as shown below.

## State EKF vs Dual EKF -SOC Estimation in the Presence of Sensor Bias

The performance of State and Dual EKF in estimating SOC was compared. A terminal voltage bias was added to the “true” model and the generated “true” data was used by the EKF and DEKF algorithms for SOC estimation. It can be noticed that DEKF performs better in estimating SOC when compared to EKF with the presence of voltage bias. Dual EKF has approximately 2% improvement in SOC estimation when comparing their root mean square values as seen in table 1.



Table 1 – State EKF vs DEKF Indices with Sensor Voltage Bias

|  |  |  |
| --- | --- | --- |
| **Index** | **State EKF** | **Dual EKF** |
|  | 0.0280 (2.80%) | 0.0116 (1.16%) |
|  | 1.0005 | 1.0003 |
|  | 0.0102 (1.02%) | 0.0102 (1.02%) |
|  | 8.53x10-6 | 8.74x10-6 |



Same as previously, a current sensor bias of 25mA was added to the “true” model and its generated data was used by both algorithms to estimate the state of charge. The figures below show that both EKF and DEKF perform well under a constant current bias. Table 2 show the performance indices which indicate almost identical performance for both.



Table 1 – State EKF vs DEKF Indices with Sensor Current Bias

|  |  |  |
| --- | --- | --- |
| **Index** | **State EKF** | **Dual EKF** |
|  | 0.0051 (0.51%) | 0.0051 (0.51%) |
|  | 0.9842 | 0.9842 |
|  | 0.0012 (0.12%) | 0.0012 (0.12%) |
|  | 1.084x10-5 | 1.0813x10-5 |

## EKF Parameter Variation

A common challenge encountered when implementing some estimation scheme like EKF is the effect of parameter mismatch, or physical variability of a specific parameter. Often, the solution to this is to apply principles of system identification through physical or simulated testing. However, this method is often time consuming and must be performed under precise testing conditions to output the most accurate parameter value possible. Additionally, operational use and wear can lead to parameters to evolve and change with continued service. To address this concern, the robustness of the estimation methodology being implemented should be reviewed to ensure that estimation failure will not occur due to minor parametric variability or system/model parameter mismatch.



Notice in the figures (???????) shown above, how the plots differ from the actual and the tuned EKF estimators. In plot(???), a series of SOC estimation trials were performed that varied each parameter by +-5% and would record the parameters that produced the maximum RMS SOC Error.

After performing a few further trials, it became clear that the predominate parameter responsible for the loss of tracking between the tuned EKF and EKF with mismatched parameters was the series resistance R0.

The reason for this behavior is hypothesized to be the large sensitivity of ohmic voltage drop in the DP Model as a function of R0 and the large input current. To validate the effect the series resistance has on the EKF performance, a series of sequential tests were performed that vary R0 by ± 20% to observe the estimated response.

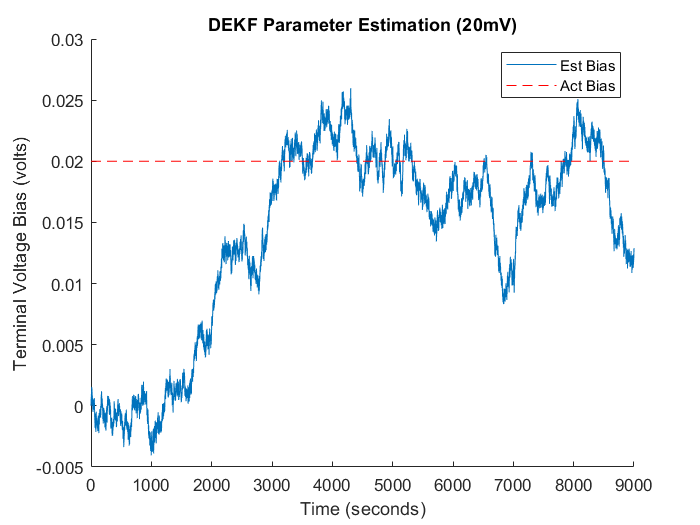
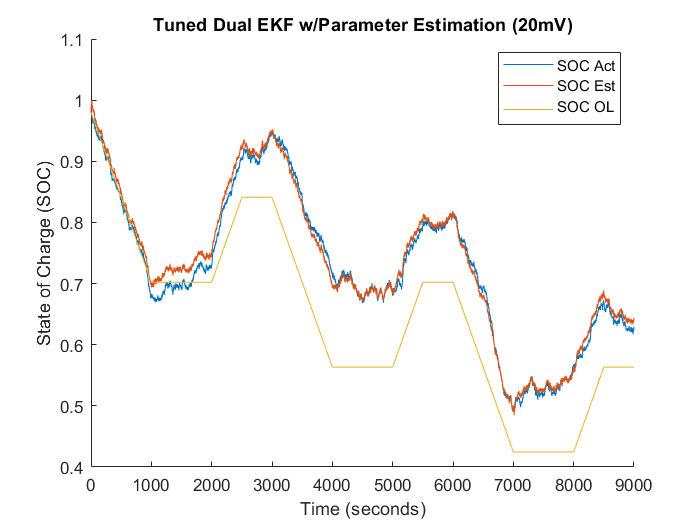
As shown in plot (????) the variability of R0 has a demonstrable effect on the non-zero current input behavior of the SOC estimator. The physical intuition behind this behavior would suggest the high input current through R0, creates an artificially voltage drop which acts like an output bias skewing the measurement update routine of the EKF algorithm.

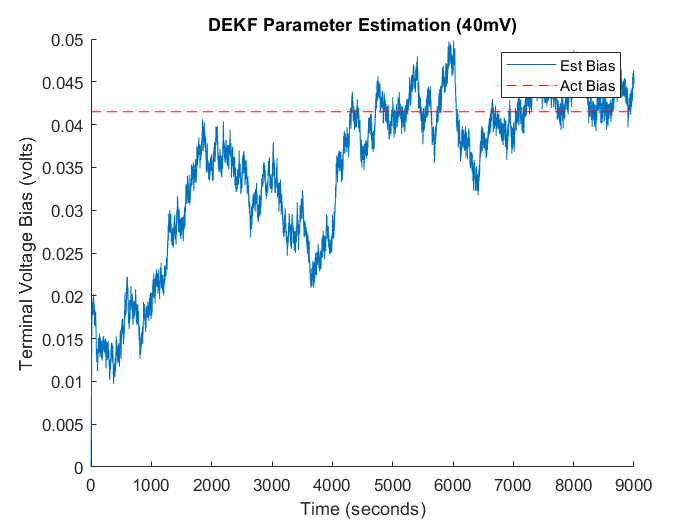
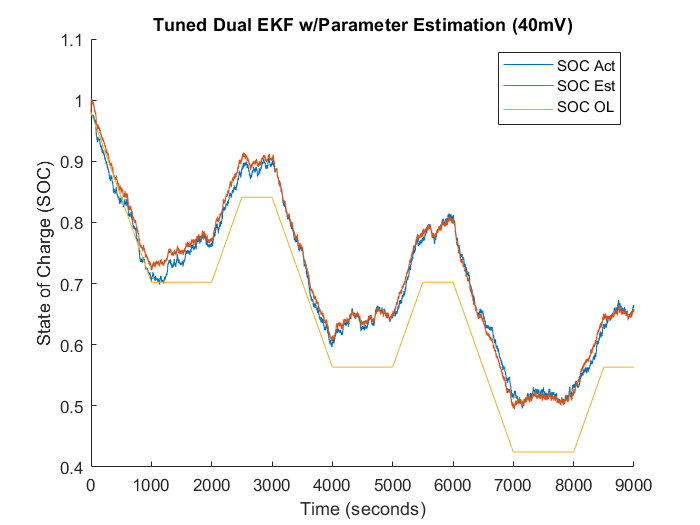
It should be further noted, that all other parameter variations aside, when R0 is tuned to match the approximate value of the true battery, the estimation results are , relatively speaking, fairly close the actual SOC.

## Sensor Bias Estimation

Voltage Bias

As the System Model and Analysis section briefly covered, every ‘real’ system exhibits some sort of noise or bias that presents the likelihood of the distorting and degrading acceptable estimation performance. To mitigate this behavior, the following plots demonstrate the benefits of implementing a tuned Dual EKF as means of estimating the SOC and the sensor bias (voltage bias). It should be noted that the ‘actual’ data used as ground truth in this simulation was reperformed with a constant 20mV. This prescribed value allows for the following comparison in plot (???) to be made revealing a general trend of the system to approximately oscillate about the actual voltage bias.





Current Bias

The results of the DEKF with current sensor bias is demonstrated below. A constant current bias of 0.25% of full scale was used from an online sensor specification data sheet. DEKF performs well in the case of constant current sensor bias with SOC average error at 0.24%. The indices

And also indicate good performance. The algorithm is able to track the bias which is modeled as random-walk. The bias is initialized at zero (no bias) as the level is generally unknown. The bias estimation error settles within 8500 seconds when tuning its fictitious noise to be very small (10-8.4).

# Future Work

Briefly summarize your project and its findings. Discuss any open questions or potential avenues for further research.

UKF, PF, Adaptive EKF, Gain Scheduled EKF, MHE.

# References

Use IEEE format for your references. It is useful but not necessary to use Word’s built in features for references and bibliographies.

|  |  |
| --- | --- |
| [1] | IEEE Periodicals, "IEEE Reference Guide," IEEE, Piscataway, NJ, 2018. |

# Supplemental Material

Include all Matlab code (Matlab has a “publish” feature that will help format your code nicely for Word). If you have Simulink models, include pictures of the models and code for any user-defined functions. If applicable, include additional figures and any other important work that you did not include in the body.

## Matlab Code

### File 1

(code here)

### File 2

(code here)

## Simulink Models

### Model 1

(image here)

(code for user-defined functions here)

## Additional Figures

## Anything Else